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A Note on Cubic Interactions in PP-Wave Light Cone String Field Theory

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Abstract

We study the string modes in the pp-wave light-cone string field theory. First, we clarify the discrepancy between the Neumann coefficients for the supergravity vertex and the zero mode of the full string one. We also repeat our previous manipulation of the prefactor for the string modes and find that the prefactor reduces to the energy difference of the cos modes minus that of the sin modes. Finally, we discuss off-shell three-string processes.

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1 Introduction

String theory on pp-wave background and $\mathcal{N} = 4$ SYM gauge theory restricted to large R charge were proposed to be dual in [1]. This proposal was partly motivated by the fact that pp-wave background can be obtained by taking the Penrose limit of AdS space [2]. The explicit comparison is made possible because string theory on the pp-wave background can be solved [3, 4] despite the fact that there exists non-zero RR flux. The anomalous dimension of certain gauge theory operators have been computed in [5, 6, 7, 8] and shown to agree with the light-cone energy of the dual string states. As for the string interaction part, the explicit proposal of the correspondence between the string theory and the gauge theory quantities [7]

$$\langle 1|\langle 2|\langle 3|H\rangle = \mu(\Delta_1 + \Delta_2 - \Delta_3)C_{123}, \tag{1.1}$$

follows from unitarity check for large μ. There have been many reports in the literature verifying this relation [9, 10, 11, 12, 13, 14, 15]. Despite the fact that all the tests for the on-shell three-point Hamiltonian matrix elements of scalar excitations have been successful, similar relation for vector excitations [16, 17] or for four-point function [18] seems to avert a naive generalization.

We address three issues of the pp-wave light-cone string field theory constructed in [19]. First of all, it has been noted in the literature [10] that the supergravity Neumann matrices do not match the zero-mode of the string Neumann matrices with $\mu \to \infty$. In this note, we resolve the origin of this discrepancy.

The role of the prefactor in the pp-wave light-cone string field theory was discussed in [7, 9, 13, 14]. Through the unitary check, the contribution of the prefactor was proposed to be just an overall factor of difference in energy of incoming and outgoing string states. In [13], the prefactor was recast in a form to make this fact manifest. However, the analysis was restricted to the supergravity vertex. In [14], the Hamiltonian matrix elements including the full string prefactor were calculated as a whole and the proposal of [7] was confirmed to first order in λ', but the explicit evaluation was only restricted to two processes and the role of the prefactor was not identified. Here we would like to combine [13] and [14] and compare the prefactor for the full string vertex with a difference in energies of string states for any $\mu\alpha' p^+$ using a factorization theorem of the Neumann coefficients shown in [20, 21].

Thirdly, let us make an attempt to extend the conjecture to energy non-preserving processes explicitly. Energy non-preserving processes are proposed [7] to correspond to non-perturbative effects in the gauge theory side. We shall show that the result of string theory cannot be reproduced only from perturbative gauge theory. We consider, at the tree-level, energy non-preserving process with two incoming states with $m+1$ and $n+1$ impurities and one outgoing state with $m+n$ impurities. On the gauge theory side it is known that the contribution is sub-leading in $1/J$ and vanishes in the pp-wave limit. However, on the string theory side the corresponding correlation function is proportional to $\bar{N}^{(12)}(\bar{N}^{(13)})^m(\bar{N}^{(23)})^n$ and is non-vanishing. This is anothor sign that the proposal of [7] should be modified for more general string interactions.

In the following sections we shall address these questions. We clarify the discrepancy between the supergravity vertex and the string vertex in the following section. In sec. 3, we repeat the manipulation of the prefactor in our previous paper [13] for the full string vertex. We also discuss the energy non-conserving process in sec. 4. Finally we conclude.

Note added

After our submission of the present paper to the hep-th archive, we were informed by the author of [21] that the formula (3.13) in the original version of the present paper, which was first obtained in [14], should be corrected by an extra factor of i on the RHS. Accordingly, (3.14) and (3.15) also have to be corrected by a minus sign. Therefore, our original claim in sec. 3, that the prefactor reduces to the energy difference, no longer holds. Instead, it reduces to the energy difference of the non-negative (cos) modes minus that of the negative (sin) modes. We have made corrections in sec. 3 accorgingly.

2 Supergravity vertex

In this section, we shall clarify the discrepancy between the supergravity vertex and the zero modes of the string vertex. In this paper, we mainly adopt the notation of [19, 14, 22]. Only in this section we set $\alpha_{(3)} = -1$ instead of introducing β or y for simplicity. In the final result, $\alpha_{(3)}$ can be restored on dimensional grounds. The Neumann coefficients for the bosonic modes are given as

$$\bar{N}^{(rs)} = \delta_{rs}1 - 2\sqrt{C_{(r)}}X^{(r)\mathrm{T}}\frac{1}{\Gamma_a}X^{(s)}\sqrt{C_{(s)}}, \tag{2.1}$$

with the matrix Γ_a

$$\Gamma_a = \sum_{r=1}^{3} X^{(r)}C_{(r)}X^{(r)\mathrm{T}}. \tag{2.2}$$

Here $X^{(r)}$ denote infinite matrices of Fourier expansion for the third string in terms of the other two with their indices running over the set of all integers. If we set $X^{(3)} = 1$, then $X^{(r)}$ $(r = 1, 2)$ can be expressed as $(r, s = 1, 2,\ \epsilon^{12} = 1)$

$$X^{(r)} = \begin{pmatrix} -(1/\alpha^{(r)})\sqrt{C}^{-1}A^{(r)}\sqrt{C} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -(\epsilon^{rs}\alpha^{(s)}/\sqrt{2})\sqrt{C}B & \sqrt{C}A^{(r)}\sqrt{C}^{-1} \end{pmatrix}, \tag{2.3}$$

with three rows(columns) representing the negative modes, the zero mode and positive modes respectively, and $A^{(r)}_{mn}$, B_m, C_{mn} and $C_{(r)mn}$ are given by

$$A^{(1)}_{mn} = (-1)^{m+n+1}\frac{2}{\pi}\frac{\sqrt{mn}\alpha_{(1)}\sin m\pi\alpha_{(1)}}{n^2 - m^2\alpha^2_{(1)}}, \tag{2.4}$$

$$A^{(2)}_{mn} = (-1)^{m}\frac{2}{\pi}\frac{\sqrt{mn}\alpha_{(2)}\sin m\pi\alpha_{(1)}}{n^2 - m^2\alpha^2_{(2)}}, \tag{2.5}$$

$$B_m = (-1)^{m+1}\frac{2}{\pi}\frac{\sin m\pi\alpha_{(1)}}{m^{3/2}\alpha^{(1)}\alpha^{(2)}}, \tag{2.6}$$

$$C_{mn} = \delta_{mn}m, \qquad C_{(r)mn} = \delta_{mn}\omega_{(r)m}, \quad r = 1,2,3 \tag{2.7}$$

with $\omega_{(r)m} = \sqrt{(\mu\alpha_{(r)})^2 + m^2}$ for $r = 1,2$ and $\omega_{(3)m} = \sqrt{\mu^2 + m^2}$.

If we take the large μ limit for the zero mode of the Neumann coefficient matrices $\bar{N}^{(rs)}_{00}$, we find that

$$\bar{N}^{(rs)}_{00} \to \begin{pmatrix} 0 & 0 & -\sqrt{\alpha_{(1)}} \\ 0 & 0 & -\sqrt{\alpha_{(2)}} \\ -\sqrt{\alpha_{(1)}} & -\sqrt{\alpha_{(2)}} & 0 \end{pmatrix}. \tag{2.8}$$

Clearly, they do not agree with the supergravity vertex M^{rs}

$$M^{rs} = \begin{pmatrix} \alpha_{(2)} & -\sqrt{\alpha_{(1)}\alpha_{(2)}} & -\sqrt{\alpha_{(1)}} \\ -\sqrt{\alpha_{(1)}\alpha_{(2)}} & \alpha_{(1)} & -\sqrt{\alpha_{(2)}} \\ -\sqrt{\alpha_{(1)}} & -\sqrt{\alpha_{(2)}} & 0 \end{pmatrix}. \tag{2.9}$$

In the construction of the supergravity vertex, the dependence of μ does not appear explicitly in the Neumann coefficient matrices M^{rs}, so one might regard this mismatch as a puzzle. However, the supergravity vertex is constructed implicitly under the assumption that the zero modes decouple completely from the higher ones. This is not true in general because there are non-vanishing overlaps between the zero modes and the positive excited modes in the Fourier expansion matrices $X^{(r)}$. It is only in the flat space limit $\mu \to 0$ that the zero modes should decouple.

Let us demonstrate this observation more explicitly. Evaluating the matrix Γ_a (2.2) by substituting the expression for $X^{(r)}$ (2.3), we find that the zero modes and the positive modes decouple as

$$\Gamma_a = \begin{pmatrix} \sqrt{C}\Gamma_-\sqrt{C} & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & \sqrt{C}\Gamma_+\sqrt{C} \end{pmatrix}. \tag{2.10}$$

Using this expression, we find*

$$\frac{\bar{N}^{(rs)}_{00}}{M^{rs}} = 1 - \mu\alpha_{(1)}\alpha_{(2)}B^{\mathrm{T}}\Gamma_+^{-1}B \equiv R, \tag{2.11}$$

*The expression (2.11) and its behavior in the flat space limit $\mu \to 0$ were also discussed in [21].

for $r, s = 1, 2$, while $\bar{N}^{(rs)}_{00}/M^{rs} = 1$, for $r = 3$ or $s = 3$. From the above observation, we expect that $R \to 1$ as we take the flat space limit $\mu \to 0$, while $R \to 0$ as $\mu \to \infty$. Before proceeding to analytical computation illustrating this behavior, let us make a few comments.

The aforementioned dependence of R on μ can be seen from numerical analysis. In Table 1, we present a numerical result for R with $\alpha_{(1)} = 1/\sqrt{2}$ and for various value of μ. The reason we take $\alpha_{(1)} = 1/\sqrt{2}$ is purely technical; we can avoid treating the indefinite forms by adopting irrational number for $\alpha_{(1)}$.

L	$\mu = 1000$	$\mu = 100$	$\mu = 10$	$\mu = 1$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.001$
10	0.0469432	0.0471652	0.0646173	0.368166	0.889787	0.988366	0.998830
20	0.0234705	0.0239095	0.0488080	0.360486	0.887990	0.988167	0.998810
30	0.0156368	0.0162841	0.0446278	0.358092	0.887414	0.988102	0.998804
40	0.0117402	0.0125833	0.0428047	0.356936	0.887132	0.988071	0.998801
50	0.00941868	0.0104431	0.0418053	0.356258	0.886966	0.988053	0.998799

Table 1: The behavior of R with $\alpha_{(1)} = 1/\sqrt{2}$ for various μ.

Note that the second term of R in (2.11) makes R deviate from 1, and this term comes from the off-diagonal part of $X^{(r)}$. Since the off-diagonal part represents the overlap between the zero modes and the positive ones, this fact confirms the reason why the two Neumann matrices do not agree; supergravity modes do not decouple from the string modes in general.

Let us proceed with the analytical computation. The asymptotic behavior of R in the limit $\mu \to \infty$ was evaluated in [22, 20]. The result is

$$R \sim \frac{1}{\pi \mu \alpha_{(1)} \alpha_{(2)}}, \tag{2.12}$$

up to some numerical factor. For completeness, let us also consider the flat space limit $\mu \to 0$. The behavior of R in this limit is easily obtained by concerning an early work of flat space light-cone string field theory [23]. In the limit $\mu \to 0$, Γ_+ reduces to the flat space one (called Γ in [23]).

$$\Gamma_+ \to A^{(1)} A^{(1)\mathrm{T}} + A^{(2)} A^{(2)\mathrm{T}} + 1. \tag{2.13}$$

Since $B^{\mathrm{T}} \Gamma^{-1} B$ was also calculated there, the behavior of R around $\mu \to 0$ simply reads

$$R \sim 1 + 2\mu \big(\alpha_{(1)} \ln \alpha_{(1)} + \alpha_{(2)} \ln \alpha_{(2)} \big). \tag{2.14}$$

In summary, we can reproduce the expected μ-dependence of R both numerically and analytically.

3 Prefactor

In this section, we repeat the manipulation of the prefactor in [13] for the full pp-wave light-cone string vertex [19, 14]. However, we will show that our previous result [13] for the supergravity modes does not hold for the full string vertex; the original expectation that the prefactor reduces to the energy difference between the incoming and outgoing string states is no longer true. Instead, it reduces to the energy difference of the non-negative (cos) modes minus that of the negative (sin) modes.

The prefactor in the oscillator basis is given [14] as $K^I \tilde{K}^J v_{IJ}(\Lambda)$, where $K = K_+ + K_-$ and $\tilde{K} = K_+ - K_-$ with

$$K_+ = \sum_{r=1}^{3} \sum_{m=0}^{\infty} F^+_{(r)m} a^\dagger_{m(r)}, \qquad K_- = \sum_{r=1}^{3} \sum_{m=1}^{\infty} F^-_{(r)m} a^\dagger_{-m(r)}, \tag{3.1}$$

and ($\alpha = \alpha_{(1)}\alpha_{(2)}\alpha_{(3)}$)

$$\begin{aligned} v^{IJ} &= \delta^{IJ} - \frac{i}{\alpha}\gamma^{IJ}_{ab}\Lambda^a\Lambda^b + \frac{1}{6\alpha^2}\gamma^{IK}_{ab}\gamma^{JK}_{cd}\Lambda^a\Lambda^b\Lambda^c\Lambda^d \\ &- \frac{4i}{6!\alpha^3}\gamma^{IJ}_{ab}\epsilon_{abcdefgh}\Lambda^c\Lambda^d\Lambda^e\Lambda^f\Lambda^g\Lambda^h + \frac{16}{8!\alpha^4}\delta^{IJ}\epsilon_{abcdefgh}\Lambda^a\Lambda^b\Lambda^c\Lambda^d\Lambda^e\Lambda^f\Lambda^g\Lambda^h. \end{aligned} \tag{3.2}$$

When one restricts to the bosonic excitation, only the third term in (3.2) contributes. In addition, from the structure of the gamma matrices [13, 9], we know that except for a relative minus sign† between the two SO(4)'s, the gamma matrix reduces to a Kronecker delta. Hence, the prefactor is given explicitly as

$$\left(\sum_{r=1}^{3} \sum_{m=0}^{\infty} F^+_{(r)m} a^\dagger_{m(r)}\right)^2 - \left(\sum_{r=1}^{3} \sum_{m=1}^{\infty} F^-_{(r)m} a^\dagger_{-m(r)}\right)^2. \tag{3.3}$$

On the other hand, the difference in energy is given by

$$P^-_1 + P^-_2 - P^-_3 = \sum_{r=1}^{3} \sum_{m=-\infty}^{\infty} \left(\omega_{(r)m}/\alpha_{(r)}\right) a^\dagger_{m(r)} a_{m(r)}. \tag{3.4}$$

Acting it on the bosonic vertex $E_a|\mathrm{vac}\rangle$ gives

$$\begin{aligned} \left(P^-_1 + P^-_2 - P^-_3\right) E_a|\mathrm{vac}\rangle &= \sum_{r,s=1}^{3} \sum_{m,n=0}^{\infty} a^\dagger_{m(r)}\left(\omega_{(r)m}/\alpha_{(r)}\right) \bar{N}^{(rs)}_{mn} a^\dagger_{n(s)} E_a|\mathrm{vac}\rangle \\ &+ \sum_{r,s=1}^{3} \sum_{m,n=1}^{\infty} a^\dagger_{-m(r)}\left(\omega_{(r)m}/\alpha_{(r)}\right) \bar{N}^{(rs)}_{-m-n} a^\dagger_{-n(s)} E_a|\mathrm{vac}\rangle. \end{aligned} \tag{3.5}$$

†See also [17] for a recent proposal related to this relative minus sign.

Let us compare this expression (3.5) with the prefactor (3.3). We first concentrate on the positive modes. For these modes, $F^+_{(r)m}$ is given as [14]

$$F^+_{(r)} = \frac{1}{\sqrt{2}}\frac{\alpha}{\alpha_{(r)}}\sqrt{CC_{(r)}}U^{-1}_{(r)}A^{(r)\mathrm{T}}\Upsilon^{-1}B, \tag{3.6}$$

up to normalization[‡] with

$$\Upsilon = \sum_{r=1}^{3} A^{(r)}U^{-1}_{(r)}A^{(r)\mathrm{T}}, \qquad U_{(r)} = (C_{(r)} - \mu\alpha_{(r)})/C, \tag{3.7}$$

and the Neumann coefficients are shown to have the following useful factorization property [20, 21]:

$$\bar{N}^{(rs)}_{mn} = -\frac{\alpha}{R}\frac{m\bar{N}^{(r)}_m\bar{N}^{(s)}_n n}{\alpha_{(s)}\omega_{m(r)} + \alpha_{(r)}\omega_{n(s)}}, \tag{3.8}$$

with

$$\bar{N}^{(r)} = -\sqrt{\frac{C_{(r)}}{C}}U^{-1}_{(r)}A^{(r)\mathrm{T}}\Gamma^{-1}_{+}B. \tag{3.9}$$

Since we can also show the property $\Upsilon^{-1}B = \Gamma^{-1}_{+}B/R$ [20, 21], $\bar{N}^{(r)}_m$ is closely related to $F_{(r)m}$. In fact we can rewrite the factorization theorem (3.8) in terms of $F_{(r)}$ as

$$\bar{N}^{(rs)}_{mn} = -\frac{R}{\alpha}\frac{2}{\omega_{m(r)}/\alpha_{(r)} + \omega_{n(s)}/\alpha_{(s)}}F^+_{m(r)}F^+_{n(s)}. \tag{3.10}$$

Although the expression for $F^+_{(r)}$ (3.6) and the factorization theorem (3.8) were originally obtained for the positive modes, one can show that the present formula (3.10) holds also for the zero mode if the indefinite from of $\bar{N}^{(13)}_{00}$ and $\bar{N}^{(23)}_{00}$ is interpreted properly. Substituting this factorization theorem (3.10) into the energy difference (3.5) and exchanging the dummy labels (r, m) and (s, n), we find

$$\sum_{r,s=1}^{3}\sum_{m,n=0}^{\infty} a^\dagger_{m(r)}\big(\omega_{m(r)}/\alpha_{(r)}\big)\bar{N}^{(rs)}_{mn}a^\dagger_{n(s)}E_a|\mathrm{vac}\rangle = -\frac{R}{\alpha}\Bigg(\sum_{r=1}^{3}\sum_{m=0}^{\infty}F^+_{m(r)}a^\dagger_{m(r)}\Bigg)^2 E_a|\mathrm{vac}\rangle. \tag{3.11}$$

We can also repeat the above calculation for the negative modes by noting[§] [14]

$$\bar{N}^{(rs)}_{-m-n} = -\big(U_{(r)}\bar{N}^{(rs)}U_{(s)}\big)_{mn}, \tag{3.12}$$

$$F^-_{(r)} = iU_{(r)}F^+_{(r)}. \tag{3.13}$$

[‡]In [14] the overall normalization of the prefactor was fixed by comparing with the supergravity vertex M^{rs} with $r, s = 1, 2$. This normalization is reliable only when μ is small.

[§]We are grateful to A. Pankiewicz for informing us that (3.13) in the original version, which was first obtained in [14], should be corrected by an extra factor i.

Using these formulae, the Neumann coefficient matrices for the negative modes can be expressed as

$$\bar{N}^{(rs)}_{-m-n} = -\frac{R}{\alpha}\frac{2}{\omega_{m(r)}/\alpha_{(r)} + \omega_{n(s)}/\alpha_{(s)}} F^-_{m(r)} F^-_{n(s)}. \tag{3.14}$$

Therefore, the contribution from the negative modes gives

$$\sum_{r,s=1}^{3}\sum_{m,n=1}^{\infty} a^\dagger_{-m(r)}\big(\omega_{m(r)}/\alpha_{(r)}\big)\bar{N}^{(rs)}_{-m-n} a^\dagger_{-n(s)} E_a|\mathrm{vac}\rangle = -\frac{R}{\alpha}\Bigg(\sum_{r=1}^{3}\sum_{m=1}^{\infty} F^-_{m(r)} a^\dagger_{-m(r)}\Bigg)^2 E_a|\mathrm{vac}\rangle. \tag{3.15}$$

Consequently, the prefactor does not reduce to the energy difference, but reduces to the energy difference of the non-negative (cos) modes minus that of the negative (sin) modes:

$$(\text{Prefactor}) \sim (P_1^- + P_2^- - P_3^-)\big|_{\cos} - (P_1^- + P_2^- - P_3^-)\big|_{\sin}. \tag{3.16}$$

Note that the prefactor is diagonal only in the cos/sin basis, and not in the exp basis which is natural in the context of the PP-wave/SYM correspondence. Here $\sim$ means that this relation holds up to some scalar factor, because the normalization of the prefactor is still unknown. As we pointed out in the footnote of (3.6), in [14] the normalization of the prefactor was determined by comparing with the supergravity vertex M^{rs} $(r, s = 1, 2)$ and this normalization is reliable only for small μ. Therefore, the overall normalization should be fixed in another way. For example, if we simply replace M^{rs} by the zero modes of the string Neumann matrices $\bar{N}^{(rs)}_{00} = M^{rs}R$ when fixing the normalization, then the scalar factor no longer depends on μ but only on some numbers and α. To be more precise, it is necessary to determine the overall scalar factor completely without mentioning to the supergravity vertex. This issue of overall normalization constant can be circumvented by computing ratio of three-point functions as done in [10, 11].

4 Towards energy non-preserving process

Having acquired a systematic viewpoint of the prefactor, let us proceed by checking if the string/gauge correspondence holds beyond the energy conserving processes. We shall consider the process with two incoming states with $m+1$ and $n+1$ impurities and one outgoing state with $m+n$ impurities. Here we shall restrict ourselves to the zero modes and abbreviate $\bar{N}^{(rs)}_{00}$ as $\bar{N}_{rs}$. First of all, let us consider the string theory side. Using [13] and the argument of sec. 3, all we have to do is to calculate the following quantity:

$$\langle a_1^{m+1} a_2^{n+1} a_3^{m+n}\rangle, \tag{4.1}$$

with $\langle\mathcal{O}\rangle$ defined as

$$\langle\mathcal{O}\rangle \equiv \langle\mathrm{vac}|\mathcal{O}E_a|\mathrm{vac}\rangle. \tag{4.2}$$

Since $\bar{N}_{33} = 0$, a_3 cannot be contracted with itself. Therefore we have three types of terms: $\bar{N}_{11}\bar{N}_{13}^{m-1}\bar{N}_{23}^{n+1}$, $\bar{N}_{22}\bar{N}_{13}^{m+1}\bar{N}_{23}^{n-1}$ and $\bar{N}_{12}\bar{N}_{13}^{m}\bar{N}_{23}^{n}$. The combinatorial coefficient of $\bar{N}_{11}\bar{N}_{13}^{m-1}\bar{N}_{23}^{n+1}$ is obtained as follows. First of all, since a_2 is always contracted with a_3, we have $(m+n)(m+n-1)\cdots m$ ways to do this. The rest of a_3 have to be contracted with a_1, and there are $(m+1)m\cdots 3$ ways to do this. Finally, the remaining two a_1's have to be contracted by themselves uniquely. Therefore, the coefficient of $\bar{N}_{11}\bar{N}_{13}^{m-1}\bar{N}_{23}^{n+1}$ is given as

$$(m+n)(m+n-1)\cdots m\cdot(m+1)m\cdots 3\cdot 1 = \frac{(m+1)m}{2}(m+n)!. \tag{4.3}$$

Similar reasoning yields the coefficient of $\bar{N}_{22}\bar{N}_{13}^{m+1}\bar{N}_{23}^{n-1}$ to be

$$(m+n)(m+n-1)\cdots n\cdot(n+1)n\cdots 3\cdot 1 = \frac{(n+1)n}{2}(m+n)!. \tag{4.4}$$

The coefficient of $\bar{N}_{12}\bar{N}_{13}^{m}\bar{N}_{23}^{n}$ can be computed by substracting the previous two cases from the combinatoric factor of contracting all a_3 with a_1 or a_2. The coefficient of $\bar{N}_{12}\bar{N}_{13}^{m}\bar{N}_{23}^{n}$ is found to be

$$\begin{aligned}(m+n+2)(m+n+1)\cdots 3 - \frac{(m+1)m}{2}(m+n)! - \frac{(n+1)n}{2}(m+n)!\\ = (m+1)(n+1)(m+n)!.\end{aligned} \tag{4.5}$$

To summarize, the correctly normalized matrix element is given as

$$\begin{aligned}\left\langle \frac{a_1^{m+1}}{\sqrt{(m+1)!}}\frac{a_2^{n+1}}{\sqrt{(n+1)!}}\frac{a_3^{m+n}}{\sqrt{(m+n)!}}\right\rangle = \frac{(m+n)!}{\sqrt{(m+1)!(n+1)!(m+n)!}}\times\\ \left\{\frac{(m+1)m}{2}\bar{N}_{11}\bar{N}_{13}^{m-1}\bar{N}_{23}^{n+1} + \frac{(n+1)n}{2}\bar{N}_{22}\bar{N}_{13}^{m+1}\bar{N}_{23}^{n-1} + (m+1)(n+1)\bar{N}_{12}\bar{N}_{13}^{m}\bar{N}_{23}^{n}\right\}.\end{aligned} \tag{4.6}$$

Now let us turn to the gauge theory side. At the tree-level, we have

$$\langle \mathcal{O}^{J_1}_{0**(m+1)}\mathcal{O}^{J_2}_{0**(n+1)}\bar{\mathcal{O}}^{J}_{0**(m+n)}\rangle = \frac{1}{\mathcal{N}_{J,m+n}}\frac{1}{\mathcal{N}_{J_1,m+1}}\frac{1}{\mathcal{N}_{J_2,n+1}}\frac{(J_1+m)!}{J_1!m!}\frac{(J_2+n)!}{J_2!n!}, \tag{4.7}$$

with $\mathcal{N}_{J,n} = \sqrt{N^{J+n}(J+n-1)!/(J!n!)}$. To compare this result with that of the string theory side, we have to take the pp-wave limit: $J, N \to \infty$ with J^2/N fixed. In this limit, the ratio to the vacuum three-point function is given by

$$\frac{\langle \mathcal{O}^{J_1}_{0**(m+1)}\mathcal{O}^{J_2}_{0**(n+1)}\bar{\mathcal{O}}^{J}_{0**(m+n)}\rangle}{\langle \mathcal{O}^{J_1}\mathcal{O}^{J_2}\bar{\mathcal{O}}^{J}\rangle} \to \sqrt{(m+1)(n+1)}\sqrt{\frac{(m+n)!}{m!n!}}\frac{1}{J}\left(\frac{J_1}{J}\right)^{(m-1)/2}\left(\frac{J_2}{J}\right)^{(n-1)/2} \tag{4.8}$$

Therefore, in the pp-wave limit the perturbative field theory results simply vanish at the tree-level. Next order in perturbation theory would give a contribution of order λ, but due to the usual non-renormalization theorem for two and three point functions of chiral primary operators [24, 25], we do not expect any perturbative corrections to above amplitudes.

From the analysis done in sec. 2, we know that $\bar{N}_{rs}$ scales as $1/\mu$ for large μ for $r, s = 1, 2$. Hence, the string amplitude scales as half-integer power of the effective coupling λ' at small λ'. It seems difficult to reproduce this behavior in perturbative gauge theory, and in order to reproduce the string theory results, we need to include non-perturbative effects as well [22]. The similarity between the coefficient of (4.8) and that of the last term of (4.6) might be a clue for resolving this mismatch.

5 Conclusion

In this note, we have reexamined the pp-wave light-cone string field theory. In doing so, we resolved the apparent puzzle regarding the mismatch between supergravity Neumann matrices and the fully string ones. The mismatch is shown to be due to the overlap of the zero modes with the excited ones. The match is of course restored in the flat space limit $\mu \to 0$. Following this¶, we concluded that the full string prefactor does not reduce to the difference in energy between the incoming and outgoing string states. Instead, it reduces to the energy difference of the non-negative (cos) modes minus that of the negative (sin) modes. Finally, we showed that the proposal of [7] does not naively generalize to the energy non-preserving amplitudes. We expect that non-perturbative effects play an important role here.

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¶This conclusion has been changed from the original version.

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